Numerical Analysis for Partial Differential Equations presenting blow-up

by

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for

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- Aim of this course

  Time-dependent partial differential equations (PDEs) such as heat equations and wave equations are of the mathematical models that describe phenomena that change over time. On the other hand, the finite difference method (FDM) is one of the methods for approximating PDEs. Although it is simple, it has a wide range of applications and is actively applied in various fields. In this course, **1. we will explain the stability and convergence of the FDM for one-dimensional heat equations and wave equations. 2. In addition, we actually implement the FDM for heat equations and wave equations. 3. Moreover, we will explain the FDM that expresses the blow-up phenomena.** We introduce a method to obtain the blow-up time numerically and prove its convergence. In addition, we introduce a method to obtain the blow-up curve for the nonlinear wave equation numerically.
- Sessions Content
  1. Stability and Convergence of the FDM

     If the FDM does not diverge, we say the FDM is stable. Moreover, we say the FDM has a convergence if the FDM converges to PDE's solution which is the object of discretization. In this session, we introduce the FDM for one dimensional heat and wave equations and prove its stability and convergence mathematically.
  2. Implementation of the FDM

     In this session, we introduce python and its basic usage. Moreover, we implement the FDM for heat and wave equations using python. We confirm its stability and convergence numerically.
  3. Finite Difference Method for Blow-up Phenomena：

     It is difficult compute blow-up phenomena because PC can only compute finite value. In this session, we introduce a FDM which express blow-up phenomena. We prove the numerical blow-up time converges to the real blow-up time. Moreover, we introduce the numerical blow-up curve and implement it.